TAX REVENUES FORECASTING WITH INTERVENTION TIME SERIES MODELING

By Marjan Nikolov, MSc
USAID Fiscal Reform Project

The subject of this paper is to illustrate a systematic approach in time series modeling named as intervention analyses with application to tax revenues forecasting. An intervention analysis is a part of a more complex area named Multi-Equation Time Series Modeling. The application will be on the collected monthly tax revenues in the Republic of Macedonia for the period 1998:1 - 2002:7. For estimation I will use the E-Views software package. This paper is organized as follows; in the first stage, data and the casual inspection of the time series is done, then the intervention technique is presented. The modeling starts with the Box-Jenkins model selection and continue with the effects of the intervention analyses. In the end the results from the forecasting are presented. Conclusion follows.

THE DATA

Macedonian tax system for the period under observation comprises following taxes: Personal Income Tax, Profit Tax, Sales Tax/VAT, Excises and Financial Transaction Tax (FTT). The dynamics of the tax collection within the period is illustrated on year by year bases in the following Table 1. In order to prevent consistency in the number of observations, the FTT growth is on the first six months from the introduction till January 2002 over the first six months in year 2002 till July 2002. The seasonal effect might have affected the calculated growth, but at least the number of observations in the periods is consistent. For the other taxes it is the growth of the first seven months of this year collection over the first seven months in the last year tax collection because for the 2002 we have data July included.

The impact of the VAT implementation in 2000 and the growth of the revenue collection of 83.15 % subject to this tax are notable. Only the Profit Tax has an average negative growth for the period of –9.69 %. The FTT has a negative growth of – 0.61 % for the first two quarters this year compare to the last two quarters from the last year. Total revenues collection for the period are growing steadily on average 9.14 %; it reflects at most the VAT (83.15 %) and excises (41.44 %) positive shock in year 2000, representing 38.77 % of total revenue growth for the year.

	personal income tax	profit tax	sales tax / VAT	Excises	FTT	tax revenues total
1998	-	-	-	-	-	-
1999	7.25%	20.88%	2.86%	-10.63%	-	1.18%
2000	7.26%	-0.26%	83.15%	41.44%	-	38.77%
2001	-31.19%	14.85%	-0.11%	-9.81%	-	3.18%
2002	-0.02%	-7.40%	24.09%	-6.63%	-0.61%	6.76%
Average growth for the period	9.79%	-9.69%	21.95%	13.14%	-0.61%	9.14%

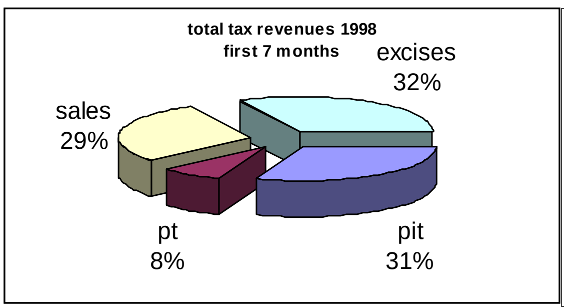
Table 1 – Tax Revenues growth year on year bases

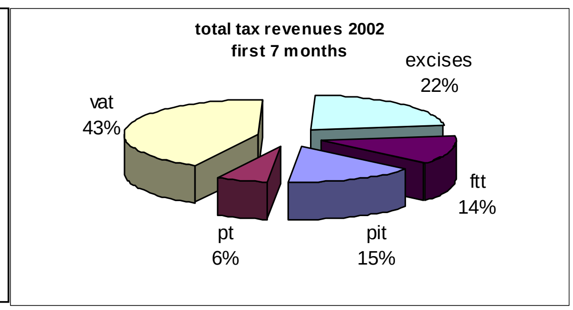
CASUAL INSPECTION OF TIME SERIES

In order to build up a tax revenues forecasting model first it is instructive to do a casual inspection of the time series on hand. What we are interested is if the series are stationary, do they have outliers, structural breaks and similar. It is of special interest to see what is the impact of the VAT implementation and FTT introduction, if it is a pulse or a pure jump shock. The intervention analyses can proceed as follows:

1. Use of Box-Jenkins ARMA model selection on the longest data span (choose between the pre or post intervention sample)[1] to find most plausible ARMA representation for the data generating process;
2. Estimate the most plausible ARMA representation for the data generating process from step 1 over the entire sample period including the effect of intervention;
3. Perform diagnostic checks on the estimated equations and do the forecasting.

Note the portfolio change in the tax structure in 1998 compared with 2002.





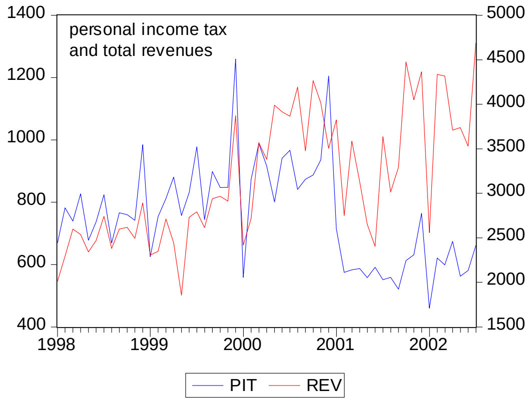
We can immediately note that personal income tax collection dropped by 16 percentage points but the VAT jumped to 43 % share in the total. Financial transaction tax was introduced 2001:7 and there were some structural changes in the tax rates of other taxes. Short comments on taxes follow after the casual inspection of the graphs illustrated below.

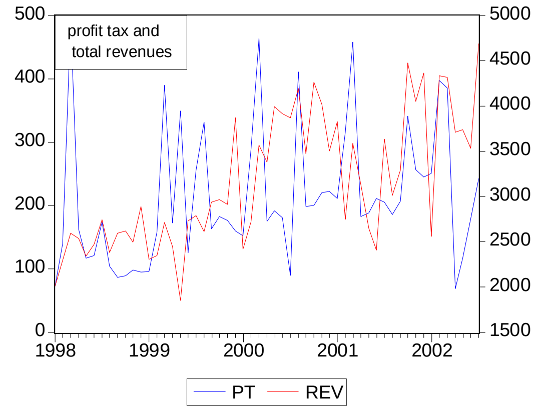
Personal income tax: structural changes 2001:2 - lower tax rates; 2001:4-2001:6 the effect of the War activities; each year in January there is a seasonal effect of collection drop in this tax and the large drop in 2001:2 from introducing lower tax rates reflects the effect of the structural break in this time series.

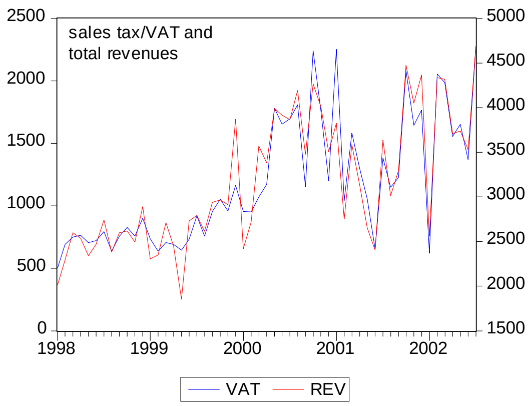
Profit tax: seasonal effect in March each year, due to the tax return filling regulations.

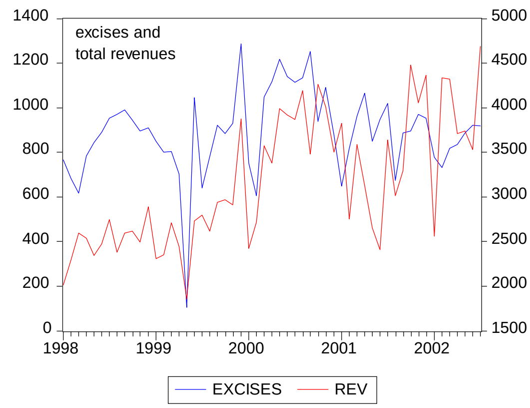
Excises' fluctuations are driving the total tax fluctuations as can be seen from the graph below; outlier-1999:5 reflecting Kosovo crises.

[1] Beside the forecasting purpose, I am interested to see what is the effect of VAT implementation so; the month of VAT implementation (2000:4) is the break between the pre and post intervention period analyses.









Sales tax/VAT: structural changes in 2000:4 - VAT implementation; 2001:4-2001:6 the effects of the War activities; 2002:1 systematic error in the calculation (outlier). The time series is more volatile with more shocks after the VAT implementation. The total tax revenue fluctuations are VAT fluctuations driven.

Financial transaction tax counts for on average 14 % of the total tax revenues for the 2002:1-2002:7 period. FTT was introduced in 2001:7.

We might consider, from the discussion above, the following dummy variables:

Pulse dummies: 2000:4 (VAT implementation); 2001:2(lower personal income tax rates); 2001:7 (FTT introduction);

Pure jump dummies: 2000:4-2002:7 (VAT implementation); 2001:7-2002:7 (FTT introduction); 2001:4-2001:6 (War activities);

Seasonal dummies: for each January in the period (personal income tax); each March in the period (profit tax);

Observation specific dummies: 1999:5 (Kosovo crises); 2002:1 (systematic error);

This casual inspection provides some useful information for the Box-Jenkins that follows.

INTERVENTION ANALYSIS

This section will select the ARMA model for the tax revenues forecasting that I believe will be the best fit to the true data generating process. The following steps will be undertaken:

1. Check for the status of either Trend Stationary (TS) or Difference Stationary (DS) time series ;
2. Identification of the data generating process using the ACF and PACF;
3. Estimation of competing ARMA models;
4. Diagnostic checking on the best model.

The time series under observation is the total monthly tax revenue collection in the Republic of Macedonia for the period 2000:4 – 2002:7[2]. Remember that I wanted to test the VAT implementation and the FTT introduction as a part of the forecasting purpose of this work. The time plot of the series is illustrated on the next Figure. Left hand graph plots the period 1998:1-2002:7 and the right hand graph is the 2000:4-2002:7 period of the total tax revenues.

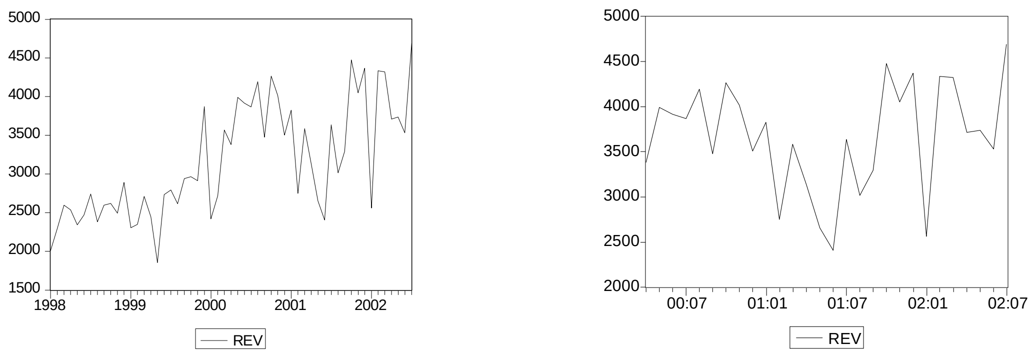


Figure - Total tax revenues

Box-Jenkins method will start with the test for stationarity performed on the time series. A clear trend and increase volatility in time in the left hand graph can be suspected. The ADF test statistic on level values (left hand graph) shows presence of a unit root. See the following table:

ADF Test	1% Critical Value	-3.5572
Level values: -2.111	5% Critical Value	-2.9167
	10% Critical Value	-2.5958
ADF Test	1% Critical Value	-3.5572
Log Difference	5% Critical Value	-2.9167
values: -9.154	10% Critical Value	-2.5958

The highly volatile appearance of the series suggests a log difference transformation in order to obtain stationarity. After the log transformation, the ADF test statistic on the log

[2] Remember to use the Box-Jenkins ARIMA model selection on the longest data span. The 2000:4 is the breaking point when the VAT was implemented. After that month we have 28 observations and before that 27 observations.

difference data (intercept included) was –9.154 thus, the null of unit root presence after this transformation was rejected at 1 % level of significance.

After the stationarity is preserved, I will proceed now to test if the time series (right hand graph) is TS or DS. The potential trend after the casual inspection of Figure 1 can be estimated from the following TS model (t is time):

$$y_t = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

The results of the estimation are illustrated in the next Table. For diagnostic comparative analysis of the three models we will look at the:

1. Significance of the estimated parameters,
2. The Ljung-Box Q-statistics for the autocorrelated residuals,
3. AIC-Akaike Information Criterion and SBC-Swartz Bayesian Criterion.

	MODEL 1	MODEL 2	MODEL 3
a_0	0.760 (0.103)	0.520 (0.409)	-0.072 (-0.318)
$a1$	-0.046 (-0.083)	-0.028 (-0.441)	0.002 (0.368)
$a2$	0.001 (0.059)	0.000 (0.474)	-
$a3$	-0.000 (-0.033)	-	-
Q(6)	15.494 (12.592)	15.408 (12.592)	15.370 (12.592)
AIC	0.091	0.010	-0.043
SBC	0.281	0.162	0.522

Notes: t-statistics in parentheses. For the Ljung-Box Q (6)-statistics $\chi^2_{0.05}$ critical values are reported in parentheses.

Table - Detrending

As we can see from the table, all parameters appear to be statistically insignificant. However, for illustration purposes I will explain the use the Ljung-Box, AIC and SBC statistic. Q (6)-statistics shows significant serial autocorrelation in the residuals for all models. That means: at least one value of the autocorrelation coefficients is statistically different from zero at the specified significance level. The AIC and SBC serve to calculate and test if additional regressor will necessarily reduce the sum of squares of the estimated residuals. Hence, the less the number of AIC and SBC the more parsimonious the model is[3]. Conclusion from this analyze is that it seems our time series under observation is DS since data do not support TS status of the data generating process.

[3] Note that SBC is asymptotically consistent while the AIC is biased toward selecting an overparametirized model.

To proceed with the time series as DS, the Autocorrelation and the Partial Autocorrelation functions of the log difference transformed series for the period 2000:4-2002:7 are presented in the next Figure. This will be the identification stage of the Box-Jenkins.

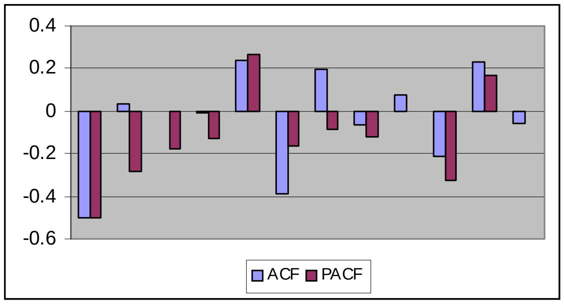


Figure – ACF and PACF for the log difference total tax revenues

From the ACF and the PACF presented in figure we can note:

1. Decaying pattern of PACF and a single large spike in the sample ACF;
2. In the ACF and PACF there are a sizeable spikes at lags 5, 6 and 10; this might suggest seasonality.
3. If we compare the sample ACF and the PACF with the theoretical ones we can suspect on MA(1), ARMA(1,1), ARMA(2,1), ARMA(2,2).

What follows next is the estimation stage of the suspected presentations of the data generating process from the note 3 above. The results are presented in the next table.

	MODEL 1	MODEL 2	MODEL 3	MODEL 4
	p=1 q=1	p=2 q=1	p=2 q=2	p=0 q=1
a_0	0.015 (0.808)	0.015 (0.789)	0.012 (0.554)	0.014 (0.784)
a_1	-0.192 (-0.596)	-0.215 (-0.480)	0.636 (3.307)	-
a_2	-	-0.023 (-0.079)	-0459 (-1.995)	-
β_1	-0.454 (-1.508)	-0.430 (-0.957)	-1.435 (-29.201)	-0.570 (-3.211)
β_2	-	-	0.995 (12.483)	-
β_{dumcar}	-0.234 (-1.406)	-0.236 (-1.358)	-0.344 (-2.948)	-0.232 (-1.355)

Note: t-statistics in parentheses.

Table - Estimation results for the ARMA (p, q) models

	MODEL 1	MODEL 2	MODEL 3	MODEL 4
	p=1 q=1	p=2 q=1	p=2 q=2	p=0 q=1
AIC	-0.403	-0.332	-0.521	-0.459
SBC	-0.213	-0.094	-0.236	-0.316
Q(12)	10.213 (15.507)	10.258 (14.067)	6.624 (12.592)	10.179 (18.307)
Jarque-Bera	0.252 (5.991)	0.257 (5.991)	0.494 (5.991)	0.195 (5.991)
ARCH	0.065 (3.841)	0.055 (3.841)	0.507 (3.841)	0.037 (3.841)

Q(12) critical value from $\chi^2_{0.05}$ are reported in parentheses. Jarque-Bera test for normality in the residuals and ARCH test for serial autocorrelation in the residuals with one lag.

Table – Diagnostics for the models

I have suggested the following competing ARMA's as a models for the true data generating process: MA(1), ARMA(1,1), ARMA(2,1), ARMA(2,2). From the graph above I am suspecting on an outlier in the month 2002:1 and because of that the observation specific dummy variable was also introduced in the models to register the effect from this outlier. From the t-statistics, the model 1 i.e. ARMA(1,1) and model 2 i.e. ARMA (2,1) appear with nonsignificant parameters at 5 % level of significance in all of the parameters. Model 3 and four are with some parameters that shows significance at 5 % level of significance. The AIC and SBC diagnostic statistics prefer model 3 and model 4 at most. Model 3 shows less possible serial correlation in the residuals and the null of the Q test was most strongly accepted. The Jarque-Bera test accepts the null of normality in the residuals in all of the models but most strongly at the model 3. Also, no ARCH was detected in the models and the null was strongly accepted in the model 4. Overall, it seams that model 3 and 4 are the best candidates from the pool. However, the t-statistic for the AR(2) parameter in model 3 is insignificant as well as the parameter for the dummy variable in the model 4. After the re estimation of model 3 without the AR(2) term and after excluding the rest insignificant parameters, the model reduced to a pure MA(1) i.e. is the model 4 but without the insignificant dummy variable. The algebraic presentation of the model is as in the following equation (t-statistics in parenthesis):

$$\Delta \log(y_t) = \underset{(0.261)}{0.003} + \varepsilon_t - \underset{(-5.724)}{0.762}\,\varepsilon_{t-1}$$

The model passes all the diagnostic checks as presented in the folowing table and will be accepted as a presentation of the true data generating process.

AIC	SBC	Q(12)	Jarque-Bera	ARCH
-0.490	-0.395	7.022 (9.488)	1.066 (5.991)	0.242 (3.841)

THE EFFECTS OF THE INTERVENTION ANALYSES

After the Box-Jenkins process resulted with a proposed model for the true data generating process, the accepted MA(1) model from the equation above will be reestimated on the entire sample span (1998:1 – 2002:7) including the pulse, pure jump, seasonal and observation specific dummies defined previously. I will use the general to specific modeling in order to choose the appropriate model. The general model will include all the dummies in the already accepted MA(1) model. Since I am interested in the VAT implementation and the FTT introduction I will estimate all variations of models with the pulse and the jump dummies on these two variables and then exclude all the statisticaly insignificant parameters. Thus, the first step is to estimate the following models as with the following FTT and VAT dummies included:

MODEL 1	MODEL 2	MODEL 3	MODEL 4	MODEL 5	MODEL 6
All dummies General model	DUM1, DUM11, DUM2, DUM22	DUM11, DUM2	DUM1, DUM22	DUM1, DUM11	DUM2, DUM22

Second, all the insignificant (by the t-statistics of the parameters) estimates beside the VAT and FTT dummies, will be excluded from the models. Third stage of the simplification search is to exclude the insignificant pulse and jump VAT and FTT parameters from the models. In the third stage the insignificant seasonal dumies are excluded from the models as well. The results from this simplification search after those three stages are presented in the following table.

	MODEL 1	MODEL 2	MODEL 3	MODEL 4	MODEL 5	MODEL 6
a_0	0.025	0.027	0.023	0.030	0.026	0.030
	(2.317)	(4.279)	(3.794)	(5.139)	(4.153)	(5.139)
β_1	-1.266	-0.997	-0.979	-0.963	-0.961	-0.963
	(-6.723)	(-15.318)	(-39.073)	(-29.056)	(-33.425)	(-29.056)
DUM11	0.164	0.351	0.223	-	0.158	-
	(1.409)	(4.088)	(2.824)		(2.103)	
DUM 22	0.001	-	-	-	-	-
	(0.007)					
DUM 1	-0.026	-0.029	-	-	-	-
	(-1.440)	(-4.029)				
DUM 2	0.080	0.053	0.019	-	-	-
	(2.391)	(4.906)	(2.350)			
DUMWAR	-0.113	-	-	-	-	-
	(-1.888)					
DUMJAN	-0.175	-0.225	-0.185	-0.175	-0.173	-0.175
	(-2.480)	(-3.544)	(-2.878)	(-2.544)	(-2.606)	(-2.544)
DUMMAR	0.128	-	-	-	-	-
	(2.069)					
DUMMAJ	-0.147	-	-	-	-	-
	(-1.138)					
DUMCAR	-0.284	-	-	-	-	-
	(-1.675)					
DUMV	-0.230	-	-0.266	-0.241	-0.229	-0.241
	(-1.661)		(-3.881)	(-3.210)	(-3.238)	(-3.210)
AIC	-1.424	-1.125	-1.122	-1.003	-1.053	-1.003
SBC	-0.982	-0.904	-0.901	-0.855	-0.869	-0.855

Note: t-statistics in parentheses.

Explanation of the dummy variables used in the model is in the next text box.

DUM 11 is the pulse dummy variable (1 for the 2000:5).
DUM 22 is the pulse dummy variable (1 for the 2001:8).
DUM 1 is the jump dummy variable (1s for the period 2000:5 – 2002:7).
DUM 2 is the jump dummy variable (1s for the period 2001:8 – 2002:7).
DUMWAR is the jump dummy variable (1s for the period 2001:4-2001:6).
DUMJAN is the pulse dummy variable (1s for seasonally January).
DUMMAR is the pulse dummy variable (1s for seasonally March).
DUMMAJ is the pulse dummy variable (1 for the outlier in 1999:5).
DUMCAR is the pulse dummy variable (1 for the outlier in 2002:1).
DUMV is the pulse dummy variable (1 for the outlier in 2001:2).

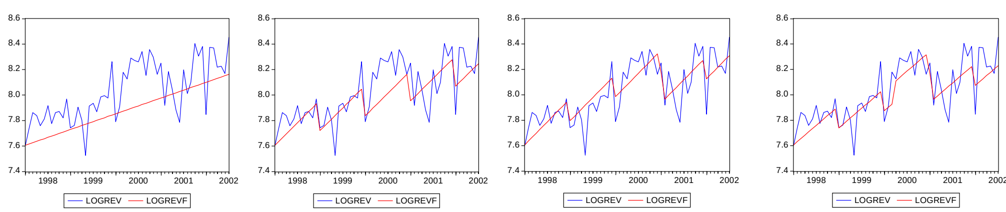
DIAGNOSTIC CHECKS

All parameters from the models (except the general one) are with significant t-statistics at 5 % significant level. Model 4 and model 6 are the same. Model 4 and 6 are the same as the model 5 if we put the restriction DUM11=0. The AIC and the SBC model selection criterion prefer model 5. By the same reasoning, model 3 (it is a model 5 with DUM2 included) is preferred to model 5. We are left to chouse between models 3 and 2. Models 2 and 3 are non-nested and I will use the J-test in order to choose the better functional form. However, the fit from both models estimated on the residuals were significant at 5 % level in both models, suggesting that another model for the sample period is needed[4]. Because of that I will return to model 5 as the next best fit model for the data generating process. Model 5 behave well regarding the diagnostic checks Q(12) (11.413 and critical value of 14.067); Jarque-Bera (2.080) and ARCH (0.087). Algebraic presentation of the model is:

$$\Delta \log(rev) = \underset{(4.153)}{0.026} + \underset{(2.103)}{0.158}\, DUM11 - \underset{(-2.606)}{0.173}\, DUMJAN - \underset{-(3.238)}{0.229}\, DUMV + \varepsilon_t - \underset{(-33.425)}{0.961}\, \varepsilon_{t-1}$$

GRAPHICAL ILLUSTRATION OF THE MODEL

The next four graphs illustrates how each variable from the last equation above affect the forecasting model and approaches to what might be best presentation of the true data generating process. From left to right first is the pure MA(1) process. Next graph illustrates the seasonally addition, the next graph adds the outlier 2001:2 effect and the last graph is with the pulse effect from the VAT implementation internalized.



[4] Since the J-test suggested a search for better model is required, I've repeated the same general to specific procedure, but instead of $\Delta \log(rev)$ with the $\Delta sqr(rev)$ transformation. In this case model 3 was accepted by the J-test compared to model 2, thus FTT jump dummy was significant at 5 % level of significance, but the variation coefficient of the forecasted errors was on average higher in this model.

CONCLUSION

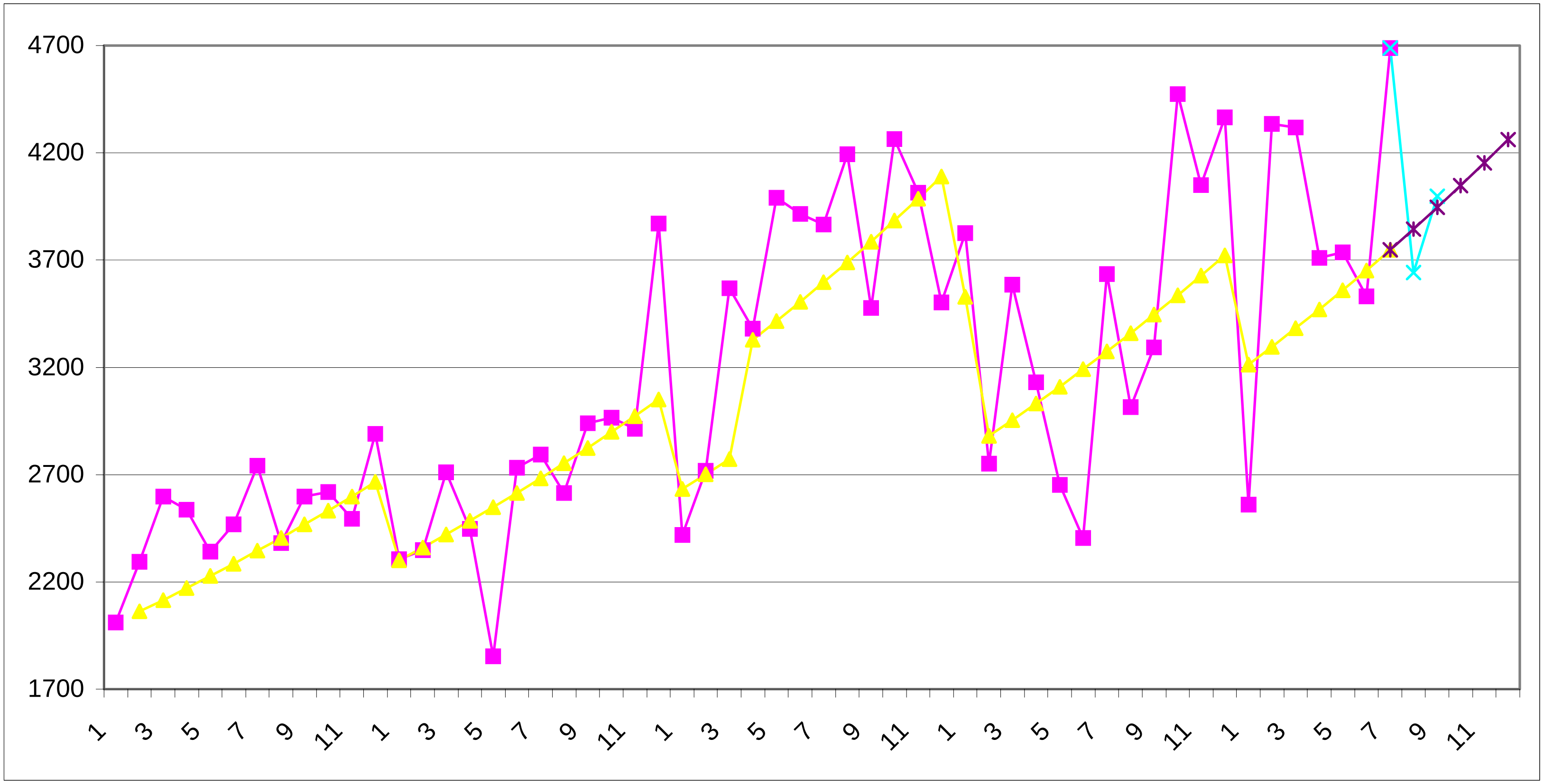
This paper illustrates tax revenue forecasting model building. The method is the time series intervention analyses. Econometric time series analysis is a powerful tool that can help in modeling economic behavior. A researcher has in disposal time to investigate more possible representation on economic behavior because software is readily available for economic laboratories and computer time is inexpensive. One area of multiequation time series modeling was illustrated by modeling the total tax revenues in Macedonia for the period 1998:1 – 2002:7. The model can be used for forecasting but only for a few time units ahead. This is because the variance of the forecasts in time series models became large in time. This is a simple, easy to use and maintain model. For more comprehensive modeling a transfer function modeling should be considered or an ECM. Time series models should be taken with precautions if the sample is with less then 50 observations. This limits tax forecasting modeling for Macedonia on quarterly or yearly data simply because the time series are too short. Because of that this monthly model for the time being might satisfy needs for a good starting point in forecasting revenues on monthly basis. It was shown that VAT affects total tax revenues as a single pulse on the process. The January seasonal effect from the personal income tax is statistically significant as well as the outlier representing introducing the lower personal income tax rates. The immediate pulse effect of the VAT implementation was 0.158 units per month increasing in the log difference of the total tax revenues. Each January the seasonally pulse effect is decreasing 0.173 units in the log difference of the total tax revenues. The pulse effect of lowering the personal income tax rate was pulse lower log difference tax revenues of 0.229 units per month. Since there is no statistically significant AR in the model all the pulse effects cause immediately effect with no delays, thus they represent the long run effect as well. However, this model couldn't find any significance in the introduction of the FTT for forecasting purposes. FTT's significance proved to be sensitive to the time series transformation in order to achieve stationarity. If the significant FTT jump dummy variable was implemented in the model (which is by econometric standards viable as from the footnote 4), the variation coefficient of the forecast errors become on average higher, thus I've decided that the model without FTT jump dummy variable is better presentation for forecasting purposes. In the next table the within the sample predictions and forecasts for August and September are presented. The August forecasts are lower for 204 mln MKD than the actual collection and the September forecasts are higher for 53 mln MKD than the actual collection.

Literature

[1] Berndt Ernst R. (1991) "The practice of econometrics: classic and contemporary", Addison-Wesley Publishing Company.
[2] Charemza Wojciech W. and Derek F. Deadman (1992) "Econometric practice", Edward Elgar Publishing Ltd.
[3] Enders Walter (1995) "Applied econometric time series", John Wiley & Sons Inc
[4] Greene William H. (1993) "Econometric analysis", Maxwell Macmillan International Publishing Group
[5] Hamilton James D. (1994) "Time series analysis", Princeton University Press.
[6] Ministry of Finance reports.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	variation coefficient	Year
							1998							**1998**
REV	2010	2294	2597	2537	2342	2469	2742	2381	2598	2619	2494	2890		2490
REVF	--	2062	2116	2171	2227	2285	2345	2406	2468	2533	2599	2666		2345
	#VALUE!	232	481	366	115	184	397	-25	130	86	-105	224	0.076	145
							1999							
REV	2306	2349	2711	2446	1854	2733	2793	2615	2939	2966	2913	3871		2670
REVF	2300	2360	2422	2485	2549	2615	2684	2753	2825	2898	2974	3051		2644
	6	-11	290	-39	-695	118	110	-138	114	68	-61	820	0.150	27
							2000							
REV	2419	2718	3568	3381	3990	3916	3866	4193	3477	4265	4014	3504		3569
REVF	2633	2701	2771	3330	3416	3505	3596	3690	3786	3884	3985	4089		3429
	-214	17	797	52	574	411	270	503	-309	381	29	-585	0.088	140
							2001							
REV	3825	2750	3585	3131	2652	2404	3635	3015	3292	4474	4050	4366		3361
REVF	3528	2880	2955	3032	3110	3191	3274	3360	3447	3537	3629	3723		3294
	297	-130	630	99	-458	-787	361	-345	-155	937	421	643	0.084	67
							2002							
REV	2560	4334	4319	3710	3737	3531	4689	**3641**	**3998**	--	--	--		3835
REVF	3212	3296	3382	3469	3560	3652	3747	**3845**	**3945**	4047	4153	4261		3714
	-652	1038	938	241	177	-121	942	**-204**	**53**	#VALUE!	#VALUE!	#VALUE!	0.073	121
							2003							
REV	--	--	--	--	--	--	--	--	--	--	--	--		#VALUE!
REVF	3676	3772	3870	3971	4074	4180	4289	4400	4515	4632	4752	4876		4230
	#VALUE!	#VALUE!	#VALUE!	#VALUE!	#VALUE!	#VALUE!	#VALUE!	#VALUE!	#VALUE!	#VALUE!	#VALUE!	#VALUE!	#VALUE!	#VALUE!

Actual values, forecasted values and forecasted errors from the estimated ARMA mode



This figure shows predictions and forecasts till 2003:12 and the forecasts and actual data for 2002:8 and 2002:9.